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MIXING OF PARTICLES IN A CIRCULATING FLUIDIZED BED

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A phenomenological model of longitudinal mixing of particles in a circulating fluidized bed is formulated. The model allows for the main features of the process: ascending motion of particles in the core of the bed and their descending motion in the annular zone (internal circulation of the solid phase); considerable changes in the concentration of particles and in the values of the ascending and descending zones over the bed height; external circulation of the solid phase and the effect of the near-bottom fluidized bed on the process as a whole. The validity of the initial proposition is confirmed by comparison of calculated and experimental curves of mixing.

The technology of a circulating fluidized bed (CFB) is of considerable current use in industry and power engineering [1, 2]. Due to the comparatively short period of study, the main laws governing heat and mass transfer in a CFB have not been adequately researched, thus making it difficult to develop and design new large-scale apparatuses with a CFB. This is in full measure true of mixing of the solid phase, the studies of which are of great practical interest for processes where continuous treatment of particles (drying, firing, combustion, etc.) is carried out or these particles gradually change their properties and need replacement (poisoning of the catalyst). Moreover, the character of mixing of particles due to their thousand-fold higher heat capacity per unit volume as compared to a gas determines the mechanism of heat transfer and equalization of temperatures in the apparatus.

By virtue of the existing [1] features of the structure of a CFB and its internal hydromechanics (substantial nonuniformity of the concentration of particles over both the height of the riser and its horizontal cross section, intense internal circulation of the solid phase, etc.), the process of mixing of the particles in this system is rather difficult for experimental investigation and for mathematical modeling. At present, only fragmentary data on the regularities of the process are available in the literature; these data are insufficient for quantitative, and often qualitative, evaluation of the influence of different factors on the intensity of mixing. The central problem with the studies is a correct interpretation of the obtained experimental data, which is directly connected with the rational choice of a physical model of the process.

The simplest one-zone model with a single parameter – the coefficient of effective longitudinal dispersion of particles — was used in [3] for analysis of experimental curves of distribution of the times of residence of particles in a CFB with a diameter of 0.152 and 0.305 m. The two-parameter model which involves the velocity of particles and the coefficient of longitudinal dispersion was used in [4] for analysis of the curves of wash-out of a tracer from a CFB with a diameter of 0.14 m. Bai et al. do not give recommendations on determination of the velocity of the solid phase. A more complex two-parameter, two-dimensional (along the coordinates r and x) model which allows for the real structure of particle fluxes in a CFB (ascending motion in the bed core and descending motion near the riser walls) and the radial dispersion of labeled (tagged) particles was used in [5]. The model considers the particular case of a constant concentration of particles over the riser height which limited the range of its use. In [6], a rather complex multiparametric circulation model of mixing, which allows for the two-zone structure of a CFB, is proposed. A substantial disadvantage of the model is an incorrect representation of diffusion and exchange terms which do not disappear at large times when the process of mixing ceases and $c_1 = c_2 = c_\infty$. It should be emphasized that this, to the same extent, is true of the models mentioned above where the form of representation of diffusion terms follows from the Fick law for systems with a constant density. Since, as is well known, a CFB is a system in which the density

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substantially changes in both the horizontal and vertical directions, the fact mentioned limits the applicability range of these models.

The present work is aimed at formulating a simple and rather universal model of mixing of particles in a CFB, which allows for the most important features of the structure and hydrodynamics of the bed and involves a minimum number of parameters to be determined. The main assumptions which form the basis for the model are as follows:

1. In the central part of the bed (the core), the ascending motion of particles at a velocity u_1 and descending motion in the annular zone at a velocity u_2 form the internal circulation of the solid phase (Fig. 1). The following formulas are used to calculate these velocities [7, 8]:

$$u_1 = u - u_t \, , \tag{1}$$

$$u_2 = 0.1\,(u - u_t)\,\mathrm{Fr}_t^{-0.7} \, . \tag{2}$$

As is seen, the velocities u_1 and u_2 are constant over the bed height.

2. The existence of the loop of external circulation of the solid phase that is produced by the particle flux J_s escaping from the upper part of the riser and then returning to the bed base (Fig. 1) is taken into account.

3. In each horizontal cross section of the riser, the equality

$$J_s = A\rho_1 u_1 - B\rho_2 u_2 \, , \tag{3}$$

holds; this equality determined the value of the specific circulation particle flux J_s which is constant over the bed height and determines the intensity of external circulation of the solid phase.

4. The local concentrations of particles in the core (ρ_1) and the annular zone (ρ_2) are interrelated as

$$\rho_2 = n\rho_1 \, , \tag{4}$$

where n is a constant coefficient. From the data of [9], $n \simeq 2$–3.

5. The mean (over the horizontal cross section of the riser) density of the bed $\rho = A\rho_1 + B\rho_2$ is variable over the height and is described by an empirical formula [10]:

$$\frac{\rho}{\rho_s} = \bar{J}_s (x')^{-0.82} \, , \ \ H_0' \le x' \le 1 \, . \tag{5}$$

6. The relative fractions of the bed core (A) and the annular zone (B) change with height; in this case, in any horizontal cross section of the bed we have

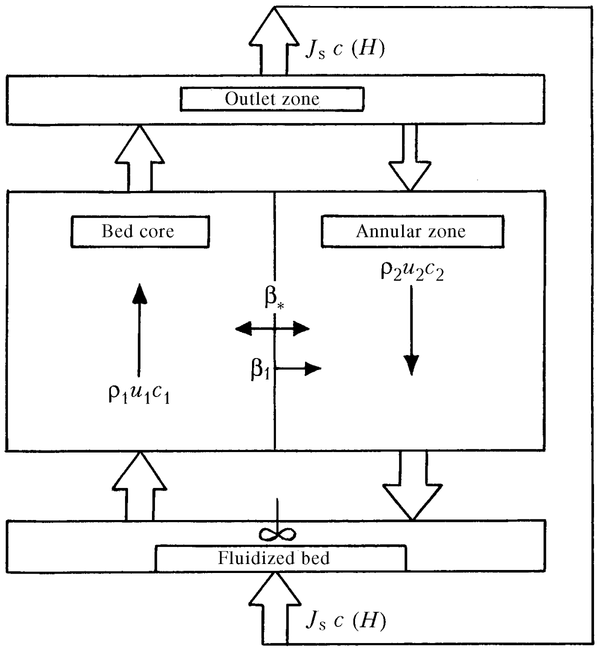
$$A + B = 1 \, . \tag{6}$$

Formulas to calculate A and B can easily be obtained on the basis of (3)–(6):

$$A = n\,\frac{u_2' + (x')^{0.82}}{u_1' + nu_2' - (x')^{0.82}\,(1 - n)} \, , \tag{7}$$

$$B = \frac{u_1' - (x')^{0.82}}{u_1' + nu_2' - (x')^{0.82}\,(1 - n)} \, . \tag{8}$$

7. The zone with a constant density and ideal mixing of particles – the near-bottom fluidized bed – exists in the lower part of the bed (Fig. 1). Its height is calculated from the formula [11]:



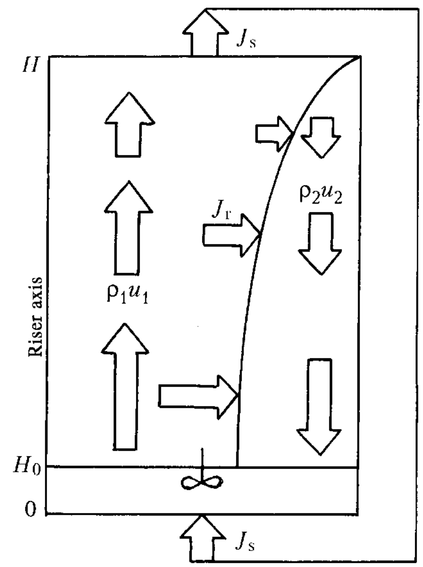


Fig. 1. Model of mixing of particles in a CFB.

Fig. 2. Schematic of particle fluxes in a CFB.

$$H_0' = 1.25\,\mathrm{Fr}_t^{-0.8}\,\bar{J}_s^{1.1}\,. \tag{9}$$

From the data of [12], the porosity of the fluidized bed slightly depends on the velocity of the gas and is a rather stable quantity. In [11], it is suggested to determine it from the expression

$$\varepsilon_{fb} = 1 - 0.33\,\mathrm{Fr}_t^{-0.045}\,. \tag{10}$$

8. The exchange of labeled particles is carried out between the bed core and the annular zone. The coefficient of exchange β_* is taken to be independent of the vertical coordinate x.

9. In both the ascending and descending zones, in addition to convective transfer, we have dispersion transfer of labeled particles with the coefficients D_1 and D_2, respectively.

10. Changes in CFB characteristics in the horizontal direction are neglected.

First, we write the continuity equations for the solid-phase flows in the bed core and the annular zone:

$$\frac{\partial A\rho_1}{\partial t} + u_1 \frac{\partial A\rho_1}{\partial x} = -A\beta_1\rho_1\,, \tag{11}$$

$$\frac{\partial B\rho_2}{\partial t} - u_2 \frac{\partial B\rho_2}{\partial x} = A\beta_1\rho_1\,. \tag{12}$$

Within the framework of the one-dimensional model, the quantity $A\beta_1\rho_1$ * allows for the existence of the radial particle flux J_r from the bed core to the annular zone (Fig. 2); this flux provides the experimentally observed decrease in the densities ρ_1 and ρ_2 with height at virtually constant values of the velocities u_1 and u_2. Having summed up (11) and (12), with account for (3) we obtain the continuity equation for the flow of external circulation of the solid phase

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* In this case, the specific form of this quantity is not of fundamental importance, since in what follows we use only the equality $A\beta_1\rho_1 = -u_1 \frac{\partial A\rho_1}{dx}$, which follows from (11) under steady-state conditions.

$$\frac{\partial\rho}{\partial t}+\frac{\partial J_s}{\partial x}=0\,, \tag{13}$$

which leads to the constancy of J_s under steady-state conditions.

With account for the assumptions made, we formulate the system of equations describing the longitudinal mixing of particles in the CFB riser:

the bed core

$$\frac{\partial A\rho_1 c_1}{\partial t}+u_1\frac{\partial A\rho_1 c_1}{\partial x}=\frac{\partial}{\partial x}\left(A\rho_1 D_1\frac{\partial c_1}{\partial x}\right)+\beta_*\rho\,(c_2-c_1)-A\rho_1\beta_1 c_1\,, \tag{14}$$

the annular zone

$$\frac{\partial B\rho_2 c_2}{\partial t}-u_2\frac{\partial B\rho_2 c_2}{\partial x}=\frac{\partial}{\partial x}\left(B\rho_2 D_2\frac{\partial c_2}{\partial x}\right)+\beta_*\rho\,(c_1-c_2)+A\rho_1\beta_1 c_1\,. \tag{15}$$

The form of the diffusion terms in (14) and (15) corresponds to the Fick law in a medium with a variable density [13]:

$$j_i=-\rho_i D_i\frac{\partial c_i}{\partial x}\,,\;\; i=1,2\,. \tag{16}$$

The contribution of these terms is most likely estimated by the quantity $1/\widetilde{\mathrm{Pe}}=\tilde{D}/(u-u_t)H$, where $\tilde{D}$ is a coefficient which has the order of D_1 and D_2. With account for $\tilde{D}\cong 10^{-3}\ \mathrm{m}^2/\mathrm{sec}$ [1, p. 345], for $1/\widetilde{\mathrm{Pe}}$ we obtain the estimate $1/\widetilde{\mathrm{Pe}}\simeq 0.2\cdot 10^{-4}$ for $H=10$ m and $u-u_t=5$ m/sec, which indicates that the fraction of the diffusion terms in (14) and (15) is negligible. With account for this fact and the continuity equations (11) and (12), system (14), (15) can be represented as follows:

$$A\rho_1\frac{\partial c_1}{\partial t}+A\rho_1 u_1\frac{\partial c_1}{\partial x}=\beta_*\rho\,(c_2-c_1)\,, \tag{17}$$

$$B\rho_2\frac{\partial c_2}{\partial t}-B\rho_2 u_2\frac{\partial c_2}{\partial x}=(\beta_*\rho+A\rho_1\beta_1)\,(c_1-c_2)\,. \tag{18}$$

Despite the simplicity of the apparatus, the system of equations (17), (18) has a rich content and reflects virtually all the most important aspects of longitudinal mixing of particles in the CFB.

For further analysis, we introduce the following notation: $p=A\rho_1$, $l=B\rho_2$, $\beta=\beta_*\rho$, and $\bar{\beta}=\beta+A\rho_1\beta_1$. Eliminating in turn c_1 and c_2 from (17) and (18), we reduce these equations to the form

$$\left(1+\frac{p}{l}\frac{\bar{\beta}}{\beta}-u_2\frac{\partial}{\partial x}\left(\frac{p}{\beta}\right)\right)\frac{\partial c_1}{\partial t}+\left(\frac{p}{l}\frac{\bar{\beta}}{\beta}u_1-u_2-u_1u_2\frac{\partial}{\partial x}\left(\frac{p}{\beta}\right)\right)\frac{\partial c_1}{\partial x}+$$
$$+\frac{p}{\beta}\frac{\partial^2 c_1}{\partial t^2}+\frac{p}{\beta}(u_1-u_2)\frac{\partial^2 c_1}{\partial t\partial x}-\frac{p}{\beta}u_1u_2\frac{\partial^2 c_1}{\partial x^2}=0\,, \tag{19}$$

$$\left(1+\frac{l}{p}\frac{\beta}{\bar{\beta}}+u_1\frac{\partial}{\partial x}\left(\frac{l}{\bar{\beta}}\right)\right)\frac{\partial c_2}{\partial t}+\left(-\frac{l}{p}\frac{\beta}{\bar{\beta}}u_2+u_1-u_1u_2\frac{\partial}{\partial x}\left(\frac{l}{\bar{\beta}}\right)\right)\frac{\partial c_2}{\partial x}+$$

$$+\frac{l}{\beta}\frac{\partial^2 c_2}{\partial t^2}+\frac{l}{\beta}(u_1-u_2)\frac{\partial^2 c_2}{\partial t\partial x}-\frac{l}{\beta}u_1u_2\frac{\partial^2 c_2}{\partial x^2}=0\,. \tag{20}$$

Equations (19) and (20) are hyperbolic equations of second order. Let us consider the important particular cases:

1. *Large times.* As is shown in [14], when $t \geq 10/\beta_*$ it is admissible to neglect the terms with $\frac{\partial^2}{\partial t^2}$ and $\frac{\partial^2}{\partial t\partial x}$ in equations of the type (19) and (20). With account for this we obtain

$$\left(\rho-\frac{1}{u_1}\frac{\partial}{\partial x}(\rho D)\right)\frac{\partial c_1}{\partial t}+\left(J_{\text{s}}-\frac{\partial}{\partial x}(\rho D)\right)\frac{\partial c_1}{\partial x}=\rho D\frac{\partial^2 c_1}{\partial x^2}\,, \tag{21}$$

$$\left(\rho+\frac{1}{u_2}\frac{\partial}{\partial x}\left(\frac{\beta}{\beta}\rho D\right)\right)\frac{\partial c_2}{\partial t}+\left(J_{\text{s}}-\frac{\partial}{\partial x}\left(\frac{\beta}{\beta}\rho D\right)\right)\frac{\partial c_2}{\partial x}=\frac{\beta}{\beta}\rho D\frac{\partial^2 c_2}{\partial x^2}\,, \tag{22}$$

where $D=\frac{pl}{\rho^2}\frac{u_1u_2}{\beta_*}$. This coefficient can be treated as the coefficient of axial "Taylor" diffusion existing in systems with a nonuniform field of longitudinal velocities and mass exchange in the transverse direction [15]. Equations (21) and (22) are the parabolic equations of unsteady-state convective diffusion with variable coefficients of dispersion D and $\frac{\beta}{\beta}D$ in the bed core and the annular zone, respectively.

2. *The steady-state mode of mixing* is realized, as is well known, with constant supply and removal of labeled material. This case is described by the following equations:

$$\left(J_{\text{s}}-\frac{\partial}{\partial x}(\rho D)\right)\frac{\partial c_1}{\partial x}=\rho D\frac{\partial^2 c_1}{\partial x^2}\,, \tag{23}$$

$$\left(J_{\text{s}}-\frac{\partial}{\partial x}\left(\frac{\beta}{\beta}\rho D\right)\right)\frac{\partial c_2}{\partial x}=\frac{\beta}{\beta}\rho D\frac{\partial^2 c_2}{\partial x^2}\,. \tag{24}$$

3. *An infinitely large coefficient of exchange* β_* for which any difference between the phases disappears. Here, generally speaking, two cases are possible:

a) u_1, $u_2 \to \infty$, but $\lim\limits_{\beta_*\to\infty}\frac{plu_1u_2}{\rho^2\beta_*}=D<\infty$; in this case, system (19), (20) is reduced to the single equation

$$\rho\frac{\partial c}{\partial t}+\left(J_{\text{s}}-\frac{\partial}{\partial x}(\rho D_\infty)\right)\frac{\partial c}{\partial x}=\rho D_\infty\frac{\partial^2 c}{\partial x^2}\,, \tag{25}$$

which is the equation of convective diffusion with a variable coefficient of dispersion D_∞ in a medium with variable density ρ;

b) u_1, $u_2 < \infty$; then $D_\infty = 0$ and it follows from (25) that

$$\rho\frac{\partial c}{\partial t}+J_{\text{s}}\frac{\partial c}{\partial x}=0\,. \tag{26}$$

Equation (26) describes the convective transfer of labeled particles at a velocity J_{s}/ρ.

4. *Absence of exchange between the phases* ($\beta_* = 0$). In this case,it is as if the phases are "quasi-isolated" and only the unilateral transfer of a labeled admixture from the core of the bed to its periphery is carried out by the flux J_r (Fig. 2). The initial system (17), (18) takes on the form

$$\frac{\partial c_1}{\partial t} + u_1 \frac{\partial c_1}{\partial x} = 0 \,, \tag{27}$$

$$\frac{\partial c_2}{\partial t} - u_2 \frac{\partial c_2}{\partial x} = \frac{p}{l} \beta_1 (c_1 - c_2) \,. \tag{28}$$

Equations (27) and (28) describe the convective transfer of labeled particles upward at a velocity u_1 (bed core) and downward at a velocity u_2 (annular zone).

System (17), (18) was used for numerical modeling of the process of mixing of a labeled admixture introduced into the near-bottom fluidized bed at the initial instant of time (Fig. 1). Such an introduction of labeled particles is most often used in experiments. The corresponding boundary-value problem has the form

$$\frac{\partial c_1}{\partial t} + u_1 \frac{\partial c_1}{\partial x} = \frac{\beta}{p} (c_2 - c_1) \,, \tag{29}$$

$$\frac{\partial c_2}{\partial t} - u_2 \frac{\partial c_2}{\partial x} = \frac{\bar{\beta}}{l} (c_1 - c_2) \,. \tag{30}$$

The boundary conditions (unambiguity conditions) are as follows:

initial conditions

$$c_1(0, x) = c_2(0, x) = 0 \,, \quad c_1(0, H_0) = c_0 \,,$$

boundary conditions

$$x = H: \quad c_1 = c_2 = c \,,$$

$x = H_0$:

$$\text{a)} \;\; t \le T: \quad \rho_{fb} H_0 \frac{\partial c_1}{\partial t} + pu_1c_1 - lu_2c_2 = 0 \,; \tag{31}$$

$$\text{b)} \;\; t > T: \quad \rho_{fb} H_0 \frac{\partial c_1}{\partial t} + pu_1c_1 - lu_2c_2 = J_s c\,(t - \Delta t, H)^* \,.$$

We note that the boundary condition at $x = H$ is the corollary of the equation

$$pu_1c_1 - lu_2c_2 = J_s c \,, \tag{32}$$

which is the balance of the fluxes of labeled particles at the riser outlet provided that there is good mixing of particles in the outlet zone (Fig. 1). The quantities p and l which enter into (29)–(31) are related to the mean density of the bed $\rho = p + l$. Allowing for this fact, from (4), (7), and (8) we can easily obtain formulas to calculate p and l:

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* It is assumed that all the labeled particles escaping from the riser arrive at the CFB again after the time Δt. The absence of their recirculation most likely corresponds to the condition $\Delta t = \infty$ ($T = \infty$).

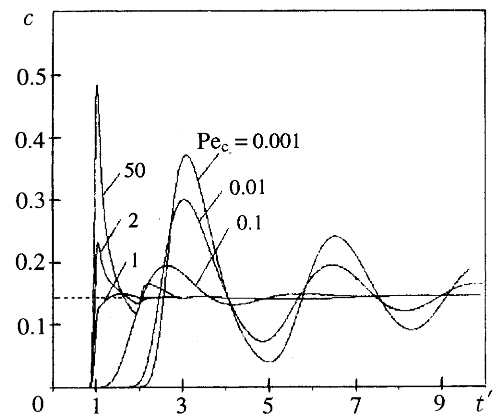

Fig. 3. Output curves of mixing for different values of the $\mathrm{Pe_c}$ number (m = 1.208, $H_0' = 0.01$, $c_\infty = 0.144$, $c_0 = 1$, $J_s = 50$ kg/(m^2·sec), $u = 6$ m/sec, $H = 12$ m).

$$p = \rho \frac{A}{A + Bn} = \rho \frac{u_2' + (x')^{0.82}}{u_1' + u_2'} , \tag{33}$$

$$l = \rho \frac{Bn}{A + Bn} = \rho \frac{u_1' - (x')^{0.82}}{u_1' + u_2'} . \tag{34}$$

We write system (29)–(31) in dimensionless form using (33) and (34):

$$\frac{\partial c_1}{\partial t'} + u_1' \frac{\partial c_1}{\partial x'} = \frac{1}{\mathrm{Pe_c}} \frac{u_1' + u_2'}{u_2' + (x')^{0.82}} (c_2 - c_1) , \tag{35}$$

$$\frac{\partial c_2}{\partial t'} - u_2' \frac{\partial c_2}{\partial x'} = \frac{1}{\mathrm{Pe_c}} \frac{u_1' + u_2'}{u_1' - (x')^{0.82}} (c_1 - c_2) . \tag{36}$$

The boundary conditions are

$$c_1(0, x') = c_2(0, x') = 0 , \; c_1(0, H_0') = c_0 ,$$

$$x' = 1 : \; c_1 = c_2 = c ,$$

$$x' = H_0' :$$

a) $t' \le T'$:

$$m H_0' \frac{\partial c_1}{\partial t'} + \frac{u_2' + (H_0')^{0.82}}{u_1' + u_2'} u_1' c_1 - \frac{u_1' - (H_0')^{0.82}}{u_1' + u_2'} u_2' c_2 = 0 ; \tag{37}$$

b) $t' > T'$:

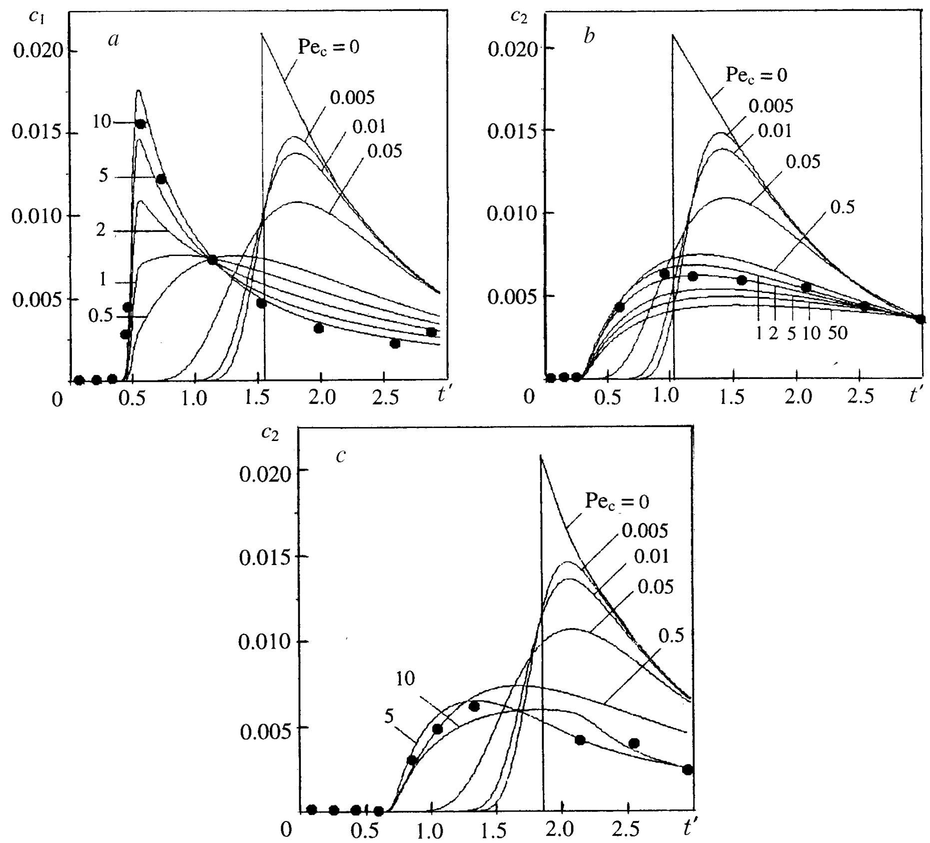

Fig. 4. Comparison of the calculated curves of mixing with the experimental data from [16]: a) $x' = 0.55$; b) $x' = 0.32$; c) $x' = 0.75$; points, experiment [16] ($m = 1.6$, $H_0' = 0.074$, $c_0 = 0.021$, $J_s = 147$ kg/(m^2·sec), $u = 4.57$ m/sec, $H = 12.2$ m).

$$mH_0' \frac{\partial c_1}{\partial t'} + \frac{u_2' + (H_0')^{0.82}}{u_1' + u_2'} u_1' c_1 - \frac{u_1' - (H_0')^{0.82}}{u_1' + u_2'} u_2' c_2 = (H_0')^{0.82} c(t' - \Delta t', 1) .$$

The quantity $m = \rho_{fb}/\rho(H_0)$ is calculated from the formula $m = 0.4\,\mathrm{Fr}_t^{-0.7}$, which follows from (5), (9), and (10). As is seen, system (35)–(37) has only one unknown parameter – the coefficient of exchange β_* which enters into the numbers $\mathrm{Pe_c}$ and $\overline{\mathrm{Pe}}_c$.

The boundary-value problem (35)–(37) was solved numerically by the finite-difference method. An implicit scheme of first order of accuracy was used. The computation region $H_0' \le x' \le 1$ was subdivided into 1000 spacings. Figure 3 shows a calculation of the concentration of labeled particles at the riser outlet ($c_1 = c_2 = c$) for different values of the $\mathrm{Pe_c}$ number. For simplicity, the calculations are made for the case $\Delta t = 0$ (labeled material is immediately transferred from the point of exit from the riser to the point of re-entry). The steady-state value of the concentration c_∞ can easily be calculated from the formula

$$c_\infty = \frac{c_0}{1 + \frac{5.5}{m}\left((H_0')^{-0.18} - 1\right)} , \tag{38}$$

which follows from the equation of material balance of the labeled admixture. Figure 4 compares the data calculated at $\Delta t = \infty$ and the experimental data from [16], where the quantities c_1 and c_2 were measured at different points of the riser with a diameter of 0.305 m. The value of β_* found by the least-squares method is 0.07 sec^{-1}. We note that the curves of mixing corresponding to $\mathrm{Pe_c} = 0$ are calculated from the formula

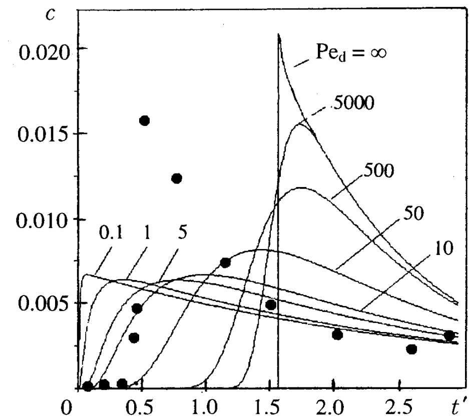

Fig. 5. Mixing curves calculated by the diffusion model for different values of the Pe_d number ($x' = 0.55$); points, experiment [16] for the concentration c_1.

$$c = c_0 \exp\left[-\frac{(H_0')^{-0.18}}{m}(t' - t_{del}')\right], \tag{39}$$

representing the solution of the equation

$$\rho_{fb} H_0 \frac{dc}{dt'} + J_s c = 0\,, \tag{40}$$

which follows from the boundary condition (a) in (31) at $c_1 = c_2 = c$. According to Eq. (26), the time of arrival of particles t_{del} at the given point of the riser is determined in this case as follows:

$$t_{del} = \frac{1}{J_s}\int_{H_0}^{x} \rho dx = \frac{5.5H}{u - u_t}\left((x')^{0.18} - (H_0')^{0.18}\right). \tag{41}$$

As is seen from Fig. 4, the calculated curves of mixing are in satisfactory agreement with the experimental data on the concentrations c_1 and c_2 obtained in [16] and allow one to correctly describe qualitative differences in the functions $c_1(t)$ and $c_2(t)$ observed in the experiment.

For comparison we considered the one-zone diffusion model with a constant coefficient of dispersion E. The form of the equation is similar to (25):

$$\rho \frac{\partial c}{\partial t} + \left(J_s - \frac{\partial}{\partial x}(\rho E)\right)\frac{\partial c}{\partial x} = \rho E \frac{\partial^2 c}{\partial x^2}\,. \tag{42}$$

The boundary conditions correspond to (31):

$$c(0, x) = 0\,,\ \ c(0, H_0) = c_0\,;$$

$$x = H:\ \ \frac{\partial c}{\partial x} = 0\,, \tag{43}$$

$x = H_0$:

$$\text{a)}\ \ t \le T:\ \ \rho_{fb} H_0 \frac{\partial c}{\partial t} + J_s c - \rho E \frac{\partial c}{\partial x} = 0\,;$$

$$\text{b) } t > T: \quad \rho_{fb} H_0 \frac{\partial c}{\partial t} + J_s c - \rho E \frac{\partial c}{\partial x} = J_s c\,(t - \Delta t, H)\,.$$

We write system (42), (43) in dimensionless form, using the expression for ρ from (5):

$$\frac{\partial c}{\partial t'} + \left((x')^{0.82} + \frac{0.82}{\mathrm{Pe_d} x'} \right) \frac{\partial c}{\partial x'} = \frac{1}{\mathrm{Pe_d}} \frac{\partial^2 c}{\partial (x')^2}\,. \tag{44}$$

The boundary conditions are

$$c(0, x') = 0\,, \quad c(0, H_0') = c_0\,;$$

$$x' = 1: \quad \frac{\partial c}{\partial x'} = 0\,,$$

$x' = H_0'$:

$$\text{a) } t' \le T': \; mH_0' \frac{\partial c}{\partial t'} + (H_0')^{0.82} c - \frac{1}{\mathrm{Pe_d}} \frac{\partial c}{\partial x'} = 0\,; \tag{45}$$

$$\text{b) } t' > T': \; mH_0' \frac{\partial c}{\partial t'} + (H_0')^{0.82} c - \frac{1}{\mathrm{Pe_d}} \frac{\partial c}{\partial x'} = (H_0')^{0.82}\, c(t' - \Delta t', 1)\,.$$

Figure 5 shows the solutions (44) and (45) obtained numerically for different values of $\mathrm{Pe_d}$ at the point $x' = 0.55$. It follows from the comparison of Figs. 4 and 5 that:

1. The diffusion model is capable of describing experimental points only at rather large times ($t' \ge 1$). At small times, the diffusion model, in contrast to the circulation one, cannot give even qualitative agreement with experimental data and describe different forms of mixing curves in the bed core and the annular zone.

2. Solutions (44) and (45) at large $\mathrm{Pe_d}$ (small coefficients E) virtually coincide with solutions (35)–(37) at small $\mathrm{Pe_c}$ (large coefficients of exchange β_*). This corresponds to the transition of Eq. (42) to (26) at small E and coincidence of the corresponding boundary conditions.

Thus, the formulated two-zone model of longitudinal mixing of particles in a CFB allows for the main features of the process and, as has been shown, is capable of satisfactorily describing experimental curves of mixing. The simplicity and thorough substantiation of Eqs. (17) and (18) make it possible to efficiently use them in practical calculations.

NOTATION

A, fraction of the horizontal cross section of the riser occupied by ascending particles (bed core); B, fraction of the horizontal cross section of the riser occupied by descending particles (annular zone); $c_1 = c_1^*/\rho_1$ and $c_2 = c_2^*/\rho_2$, dimensionless concentrations of labeled particles in the bed core and the annular zone; c_1^* and c_2^*, concentrations of labeled particles in the bed core and the annular zone; c_0, initial dimensionless concentration of labeled particles at $x = H_0$; $c = Ac_1 + Bc_2$, mean dimensionless concentration of labeled particles; $c_\infty = \lim\limits_{t\to\infty} c$; $D_\infty = \lim$ $(u_1 \to \infty$, $u_2 \to \infty$, $\beta_* \to \infty)$; D_1, D_2, and E, coefficients of dispersion of labeled particles; $\mathrm{Fr_t} = \frac{(u - u_t)^2}{gH}$, Froude number; g, free-fall acceleration; H, riser height; H_0, height of the near-bottom fluidized bed; $H_0' = H_0/H$; j_i, diffusion flow of labeled particles; J_s, circulation mass flux of particles; $\bar{J}_s = J_s/\rho_s(u - u_t)$, dimensionless mass flux of particles; $l = \beta\rho_2$; $p = A\rho_1$;

$\mathrm{Pe_c}(u-u_t)/\beta_* H$, $\mathrm{Pe_d}(u-u_t)H/E$, $\overline{\mathrm{Pe}}_c/\mathrm{Pe_c} \Big/ \left(1+0.82\frac{u_1' u_2'}{u_1'+u_2'}\mathrm{Pe_c}\frac{1}{x'}\right)$, Péclet numbers; r, radial coordinate; t, time; $t' = t(u-u_t)/H$, dimensionless time; Δt, time of recirculation (time interval between the escape of labeled particles from the upper part of the riser to the entry into its base); $\Delta t' = \Delta t(u-u_t)/H$; $t'_{del} = t_{del}(u-u_t)/H$; Δt_r, time during which particles in the bed traverse the part of the riser from $x = H_0$ to $x = H$; $T = \Delta t_r + \Delta t$, period of circulation; $T' = T(u-u_t)/H$; u, rate of filtration of the gas; u_t, free-fall velocity of a single particle; $u_1' = u_1/(u-u_t)$, $u_2' = u_2/(u-u_t)$, u_1, and u_2, velocity of particles in the bed core and the annular zone; x, vertical coordinate; $x' = x/H$; β_*, coefficient of exchange by labeled particles; β_1, coefficient introduced in (11); $\beta = \beta_* \rho$; $\overline{\beta} = \beta + p\beta_1$; ε, porosity; ρ_1 and ρ_2, density of the bed in the core and the annular zone; $\rho = A\rho_1 + B\rho_2$, bed density mean over the horizontal cross section of the riser; ρ_s, density of particles. Subscripts: 1, bed core; 2, annular zone of the bed; c, circulation model (35)–(37); d, diffusion model (44), (45); fb, fluidized bed near the gas distributor (near-bottom fluidized bed); r, radial; s, particles; t, free-fall conditions of a single particle; del, delay.

REFERENCES

1. A. P. Baskakov, in: V. G. Ainshtein and A. P. Baskakov (eds.), *Fluidization* [in Russian], Moscow (1991), pp. 333–395.
2. A. P. Baskakov, V. V. Matsnev, and I. V. Raspopov, *Boilers and Furnaces with a Fluidized Bed* [in Russian], Moscow (1995).
3. M. Rhodes, S. Zhou, T. Hirama, and H. Cheng, in: *Proc. 7th Eng. Found. Conf. on Fluidization*, Brisbane (1992), pp. 249–256.
4. D. Bai, J. Yi, Y. Jin, and Z. Yu, in: *Proc. 7th Eng. Found. Conf. on Fluidization*, Brisbane (1992), pp. 195–202.
5. C. S. Patience and J. Chaouki, in: *Proc. 8th Eng. Found. Conf. on Fluidization*, Vol. 1, Tours (1991), pp. 33–40.
6. Q. Wang, J. Zhou, J. Tu, et al., in: *Proc. 5th Int. Conf. CFBs*, Beijing (1996), pp. DB20 1–6.
7. J. Werther, E.-U. Hartge, and M. Kruse, in: *Proc. 7th Eng. Found. Conf. on Fluidization*, Brisbane (1992), pp. 257–264.
8. Yu. S. Teplitskii, *Inzh.-Fiz. Zh.*, **74**, No. 5, 177–181 (2001).
9. W. Novak, *A Comprehensive Study of the Circulating Fluidized Bed*, Thesis presented to qualify as assistant professor, Czestochowa (1989).
10. V. A. Borodulya and Yu. S. Teplitskii, in: *Proc. III Minsk Int. Forum "Heat and Mass Transfer–MIF-96,"* May 20–24, 1996 [in Russian] Vol. 5, Minsk (1996), pp. 69–74.
11. Yu. S. Teplitskii and V. I. Kovenskii, *Inzh.-Fiz. Zh.*, **74**, No. 1, 62–66 (2001).
12. F. Johnsson and B. Leckner, in: *Proc. 13th Int. Conf. on Fluidized Bed Combustion*, Orlando (1995), pp. 1–9.
13. R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena* [Russian translation], Moscow (1974).
14. Yu. S. Teplitskii, *Hydrodynamics and Heat Transfer in Free and Retarded Fluidized Beds*, Abstract of Doctoral Dissertation (in Engineering), Novosibirsk (1990).
15. G. Taylor, *Proc. Roy. Soc.*, Series A, **219**, No. 1137, 186–203 (1953).
16. R. Bader, J. Findlay, and T. M. Knowlton, in: *Proc. 2nd Int. Conf. on CFBs*, Compiegne (1988), pp. 123–138.